DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

45. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.

Solve the equation $x^3+y^2=a^2$.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Put $y=\dfrac{x(x-n^2)}{2n}$. Then we readily obtain $x^3+\left\{\dfrac{x(x-n^2)}{2n}\right\}^2=\left\{\dfrac{x(x+n^2)}{2n}\right\}^2$,

which is a general formula for finding *the sum of a cube aud a square equal to a square*, x and n representing any values. We have also the general condition, derived from the formula, $nx+y=a$. By taking $n=1$, and putting $x=$, consecutively, the natural numbers beginning with unity, we obtain a series of equations in which the consecutive values both of y and a form *the series of integral numbers the sum of any two consecutive terms of which is the square of their difference.* [Problem 43, page 370, Vol. II.]

II. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Let $y=mx$, then $x^3+m^2x^2=a^2$. $\therefore$ $x+m^2=a^2/x^2=b^2$, $\therefore$ $b^2-m^2=x$, where b and m can be any integers $b>m$. We append some values.

b	m	x	y	a
1	0	1	0	1
2	1	3	3	6
3	2	5	10	15
4	3	7	21	28
5	4	9	36	45
&c.	&c.	&c.	&c.	&c.

III. Solution by M. C. STEVENS, M. A., Department of Mathematics, Purdue University, Layfayette, Indiana.

If x be any integer and $y=\dfrac{x(x-1)}{2}$, then $x^3+y^2=\dfrac{x^4+2x^3+x^2}{4}=a^2$.

$\therefore$ $a=\dfrac{x(x+1)}{2}$. If $x=1$, then $a=1$, $y=0$. If $x=2$, then $a=3$, $y=1$, and so on.

IV. Solution by A. H. BELL, Box 184, Hillsboro, Illinois.

We write $x^3=a^2-y^2$. From the well known form

$mn=\left(\dfrac{m+n}{2}\right)^2-\left(\dfrac{m-n}{2}\right)^2$, if $x^3=mn$, the problem is answered.

Let m and n be 4 and 2; or 27 and 1; or 9 and 3; etc.; then $2^3+1^2=3^2$; $3^3+13^2=14^2$; $3^3+3^2=6^2$; etc.

V. **Solution by H. C. WILKES, Skull Run, West Virginia.**

$x^3=(a+y)(a-y)$. Let $a+y=x^2$ and $a-y=x$, then $x^2+x=2a$, and $x=\frac{1}{2}\pm\sqrt{2a+\frac{1}{4}}$. Let a be any triangular number, and from the above formula, integral values for x, a, and y can be found.

VI. **Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.**

Let $x=ky$. Then $x^3+y^2=a^2$ becomes $y^2\{k^3y+1\}=a^2$. This will be a square if $y=k^3+2$. $\therefore$ $y=k^3+2$, and $x=k(k^3+2)$ will be a solution, where k is any integer. If $k=1$, $y=3$, $x=3$ and $x^3+y^2=36$. If $k=2$, $y=10$, $x=20$, and $x^3+y^2=8100$, etc., etc.

VII. **Solution by J. H. DRUMMOND, LL. D., Portland, Maine.**

(A). If the problem is to be taken literally, $y=\sqrt{a^2-x^3}$ in which x may any number whose third power $<$ than a^2. But this does not give exact results.

(B). If it means that $x^3+y^2=\square$, let $x=my$ and we have $m^3y+1=\square=$ (say) b^2 and $y=(b^2-1)/m^3$ and $x=(b^2-1)/m^2$; but then $a=b(b^2-1)/m^3$, in which m and b may be any numbers greater than unity, but the value of a depends on x and y.

(C). By transposing $x^3=a^2-y^2$; take $x=a-y$, then $x^2=a+y$, and $a^2-2ay+y^2=a+y$, and $y=(2a+1\pm\sqrt{8a+1})/2$. As y must be less than a to make x positive, the sign of the radical term must be negative. It is readily seen that $a=n(n+1)/2$ makes $8a+1$ a square, and by reducing we get $y=n(n-1)/2$ and $x=n$, in which n may be any number.

(D). If the question means to find exact values of x and y for *any* value of a, I cannot solve it.

46. **Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.**

In $x^2+x\sqrt{xy}=a$......(1) and $y^2+y\sqrt{xy}=b$......(2) find such values of a find b as will make x and y integral; give a general solution.

I. **Solution by the PROPOSER.**

Take $y=m^2x$, and by combining the two equations and reducing we have, $\frac{b}{a}(m+1)=m^3(m+1)$ and consequently $m^3=\frac{b}{a}$.

From (1) we have $x=\pm\sqrt{\frac{a}{m+1}}$. Take $a=c^2$ and $m+1=d^2$ and substituting, we have $x=c/d$. To make this value integral, take $c=de$; then $x=e$, and $y=m^2x=e(d^2-1)^2$. But $a=c^2$, and $c=dx=de$. $\therefore$ $a=d^2e^2$; but $b=am^3=d^2e^2(d^2-1)^3$, in which a may be any whole number>1, and e any whole number.